

Network Modelling

Modern Techniques in Modelling

LONDON
SCHOOL of
HYGIENE
& TROPICAL
MEDICINE



Discuss network modelling concepts (~20 min)

Practical 1: Introduction to the igraph package (~40 min)

BREAK (~30 min)

Reed-Frost SIR Model (~10 min)

Practical 2: A network model of mpox transmission
(~40 min)

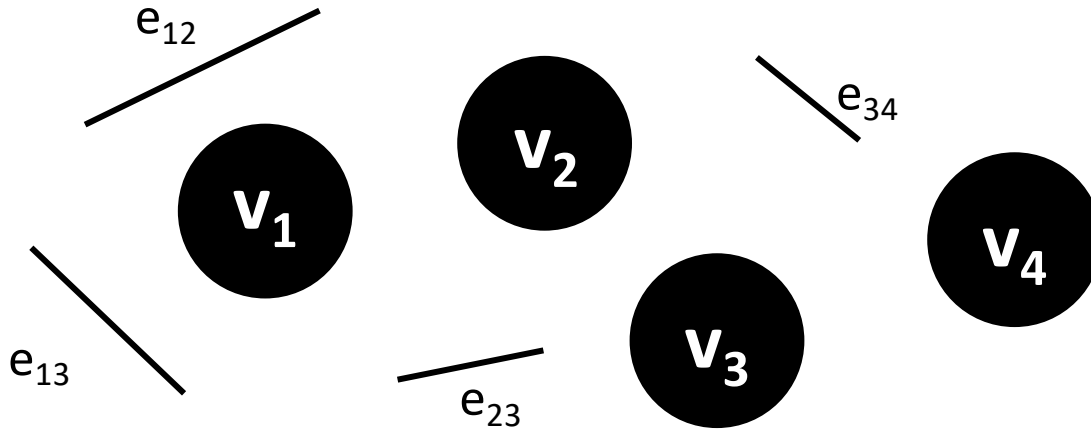
Closing discussion (~10 min)

Part 1:

What are Network Models? Why Use Networks?

What are Network Models?

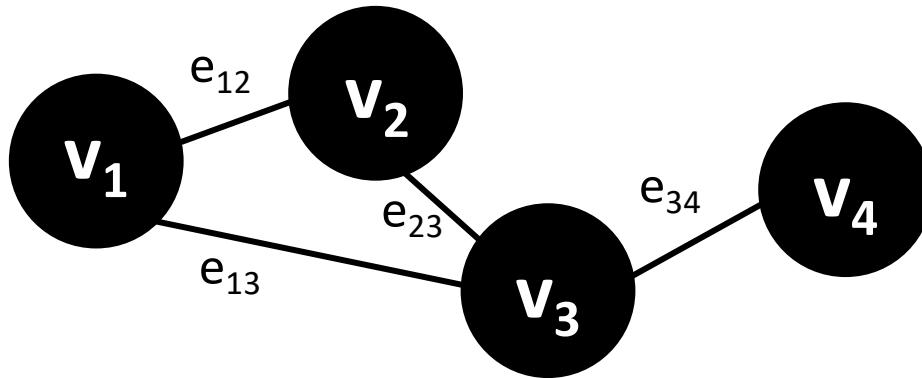
Network = set of vertices V and edges E



What are Network Models?

Network = set of vertices V and edges E

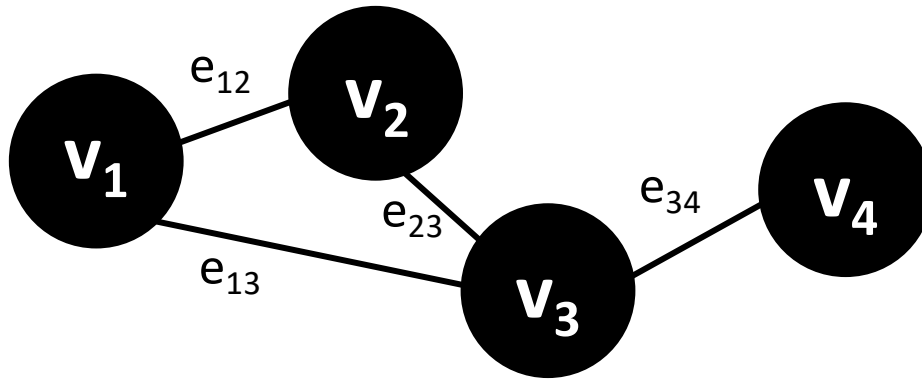
an edge e_{ij} in E connects two vertices v_i, v_j in V



What are Network Models?

Modelling: representing real phenomena by abstractions

How might networks represent infectious disease?



Some possible answers:

Generally: represent process over the *network* (e.g., transmission) + vertex *state* (e.g., SIR)

Useful to think about *contacts*:

- when transmission depends on specific contacts (e.g., STIs)
- if there is heterogeneity / structure to contacts (e.g., influenza in schools vs workplaces vs households)

Spatial relationships:

- mosquitoes moving between adjacent households
- people migrating from one region to another

Why use networks?

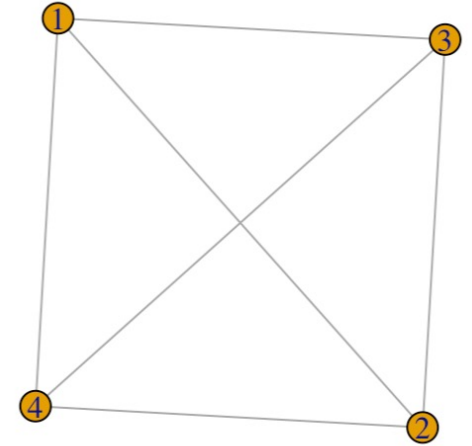
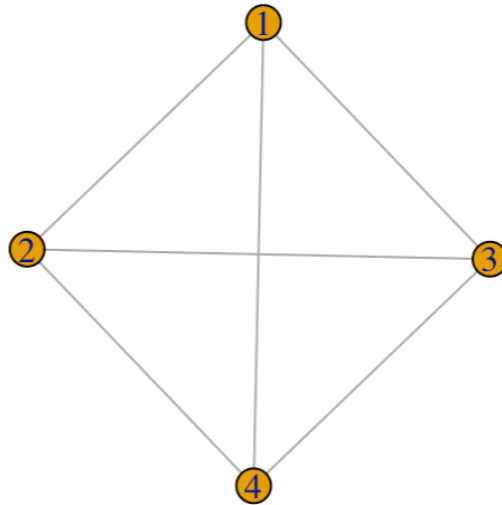
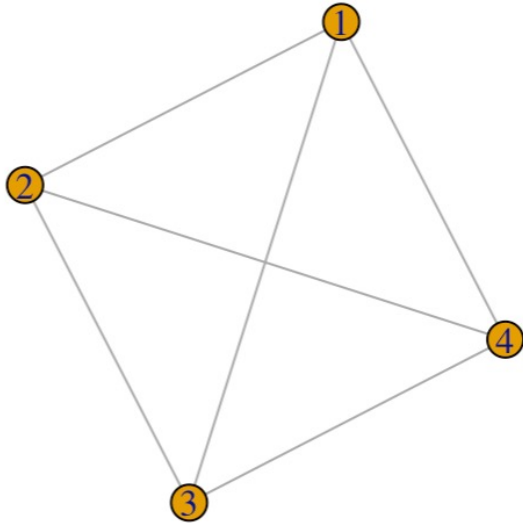
Shared features of those examples:

- “realism” aspect of the model (e.g. vs homogeneous mixing)
- clear translation between observations and model
- generalisation: e.g., why does R_0 vary between contexts? same pathogen & host - but different transmission networks

Introduction to the igraph package

Questions from the practical

1. Run the `plot(gr)` line multiple times. The plot changes. Does the graph change?

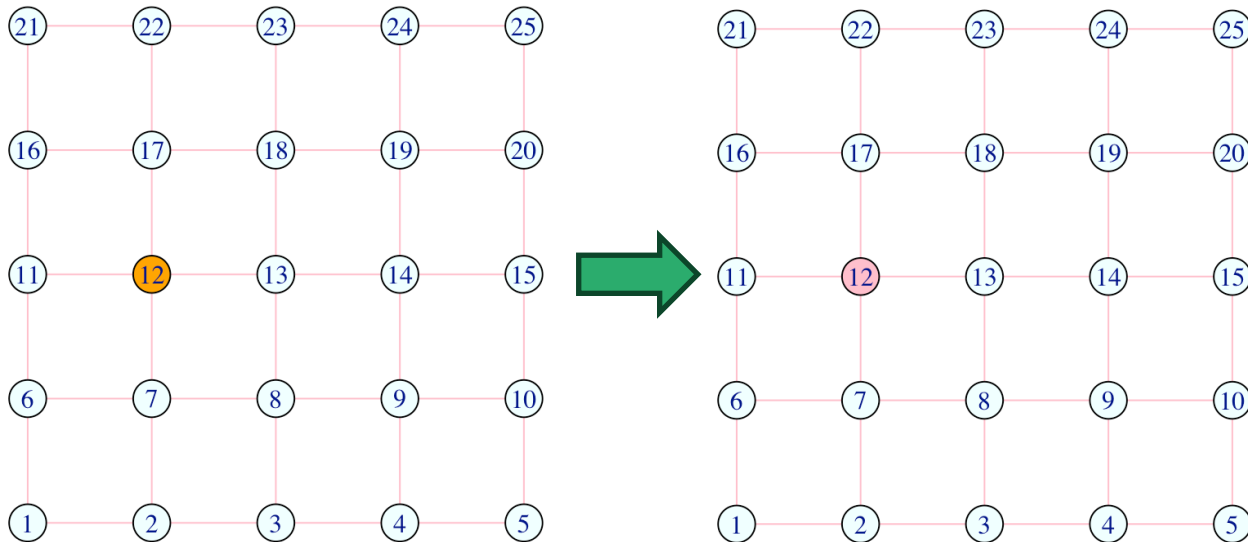


Questions from the practical

```
V(network)[12]$color <- "orange"
```

```
V(network)[color == "orange"]$color <- "pink"
```

2. What do the above lines do?



While the first line selects a specific vertex (#12), the second line selects all vertices that have `color == "orange"` — this will be useful when we use vertex properties to represent states (e.g. S, I, R).

Questions from the practical

```
v(network)[.nei(color == "pink")]$color <- "pink"  
plot(network)
```

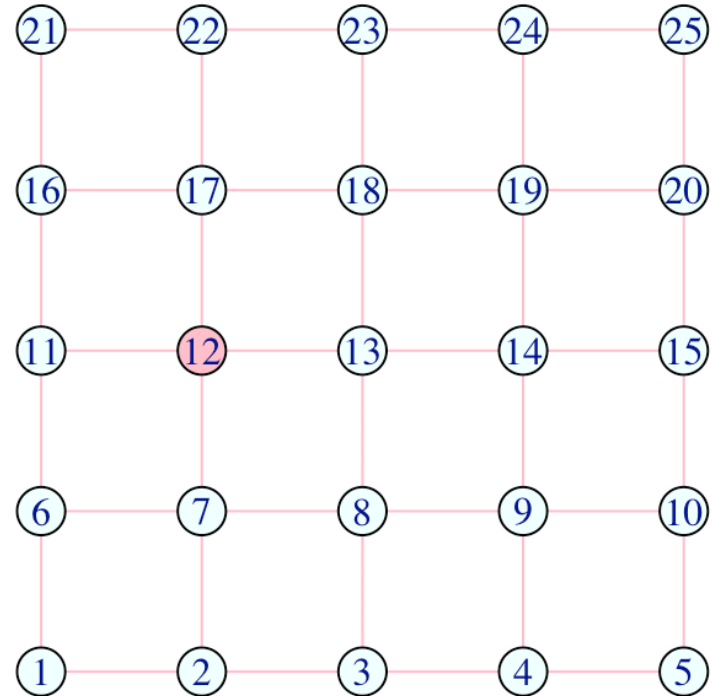
3. What is the code above doing?
What happens if you re-run the
two lines above several times?

Questions from the practical

```
v(network)[.nei(color == "pink")]$color <- "pink"  
plot(network)
```

3. What is the code above doing?
What happens if you re-run the
two lines above several times?

What relevance might selecting
neighbours of vertices have for
modelling infectious disease
transmission?



Coffee break

Part 2:

A network SIR model for mpox transmission

In this hour...

1. Describe the “Reed-Frost” model
2. Describe how this model can be implemented on a network
3. Practical: Model mpox transmission on a sexual network

The Reed-Frost model

Population, **N** , of hosts

“Time” in steps (generations): **t** , **$t+1$** , ...

Hosts either ***Susceptible***, ***Infectious***, or ***Removed***

When **I** and **S** interact at **t** ,

probability **p** that the **S** becomes **I** at **$t+1$**

(i.e., stochastic infection)

All **I** hosts at a time step **t** become **R** at **$t+1$**

This can be implemented on a network...

Population, N , of hosts

$N = 11$

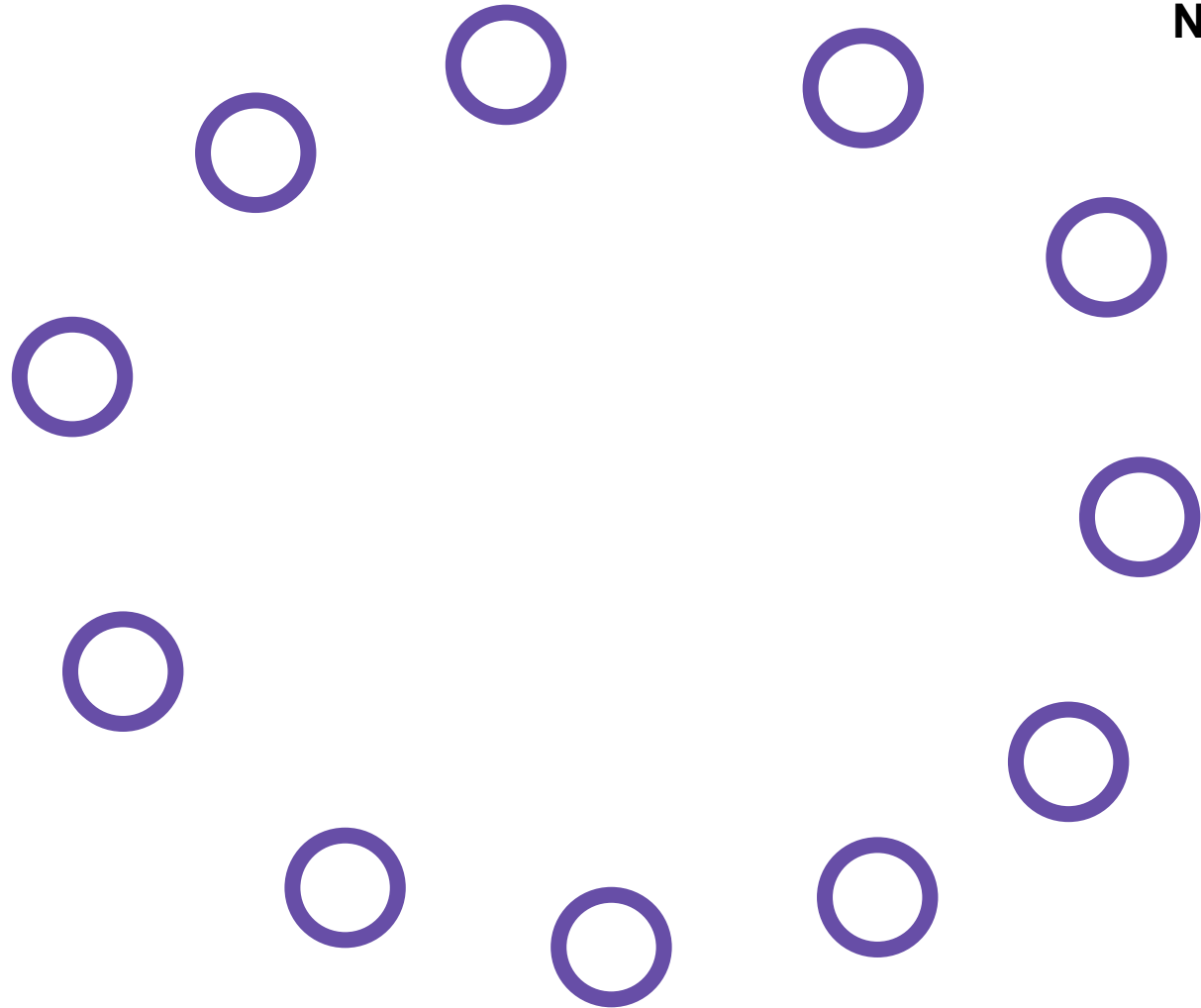
“Time” in steps: $t, t+1, \dots$

Hosts either *Susceptible*,
Infectious, or *Removed*

Each time step, *all* hosts
interact

When I and S interact at t ,
probability p that the S
becomes I at $t+1$

All I hosts at a time step t
become R at $t+1$



Population, N , of hosts

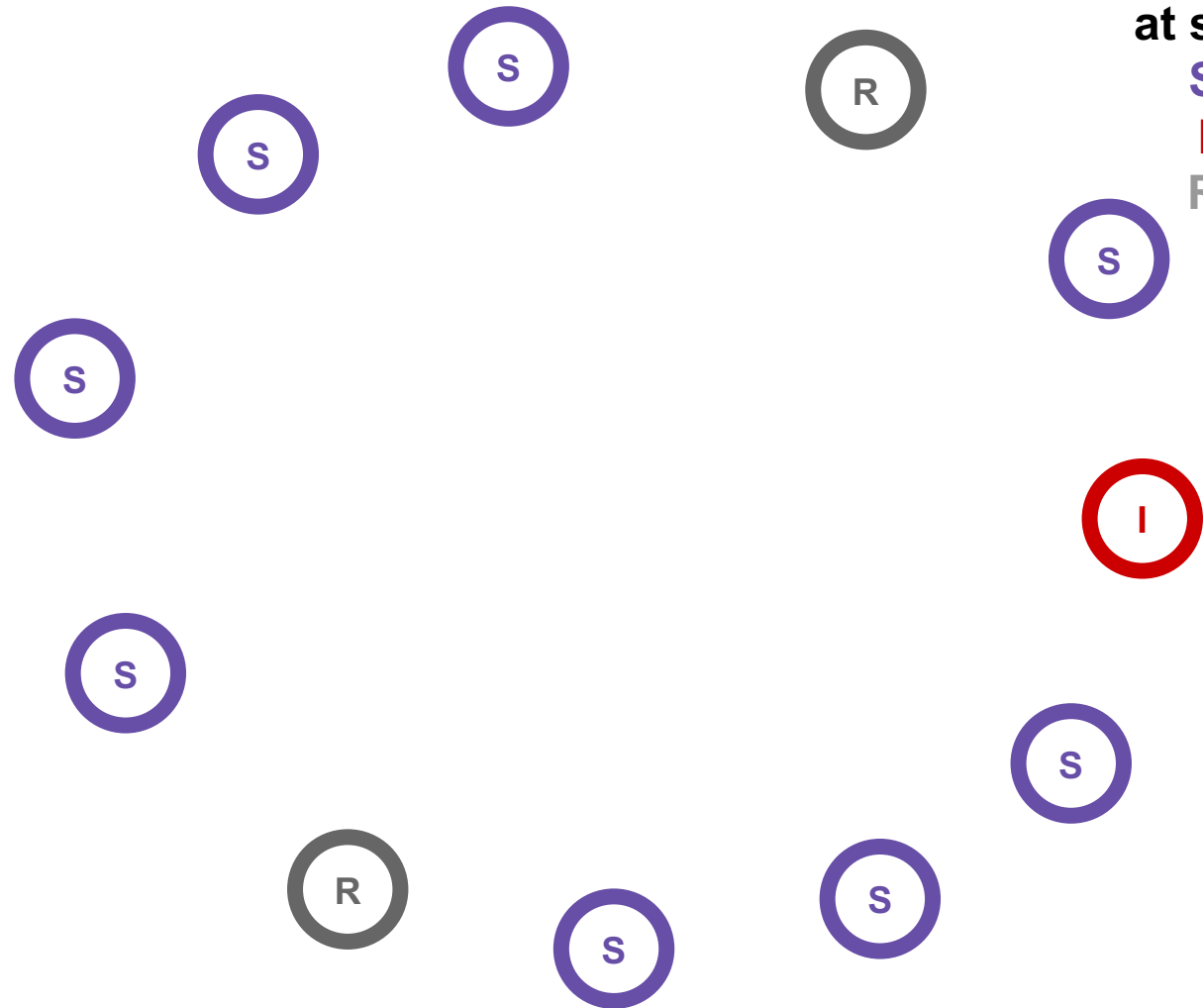
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All I hosts at a time step t
become R at $t+1$



at some t ,

$$S = 8$$

$$I = 1$$

$$R = 2$$

Population, N , of hosts

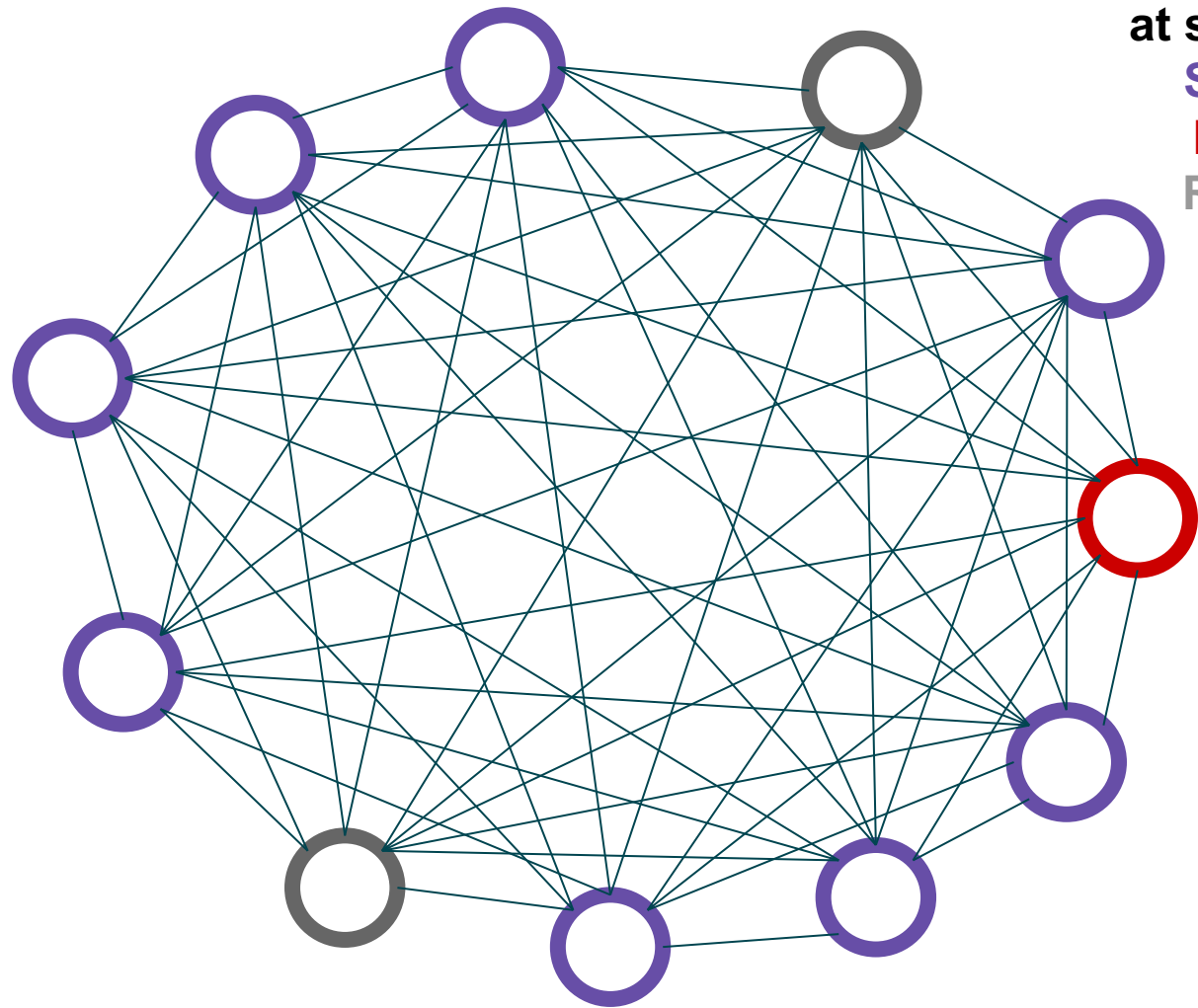
“Time” in steps: $t, t+1, \dots$

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Each time step, **all
neighbours interact**

When I and S interact at t ,
probability p that the S
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All I hosts at a time step t
become R at $t+1$



at some t ,

$S = 8$

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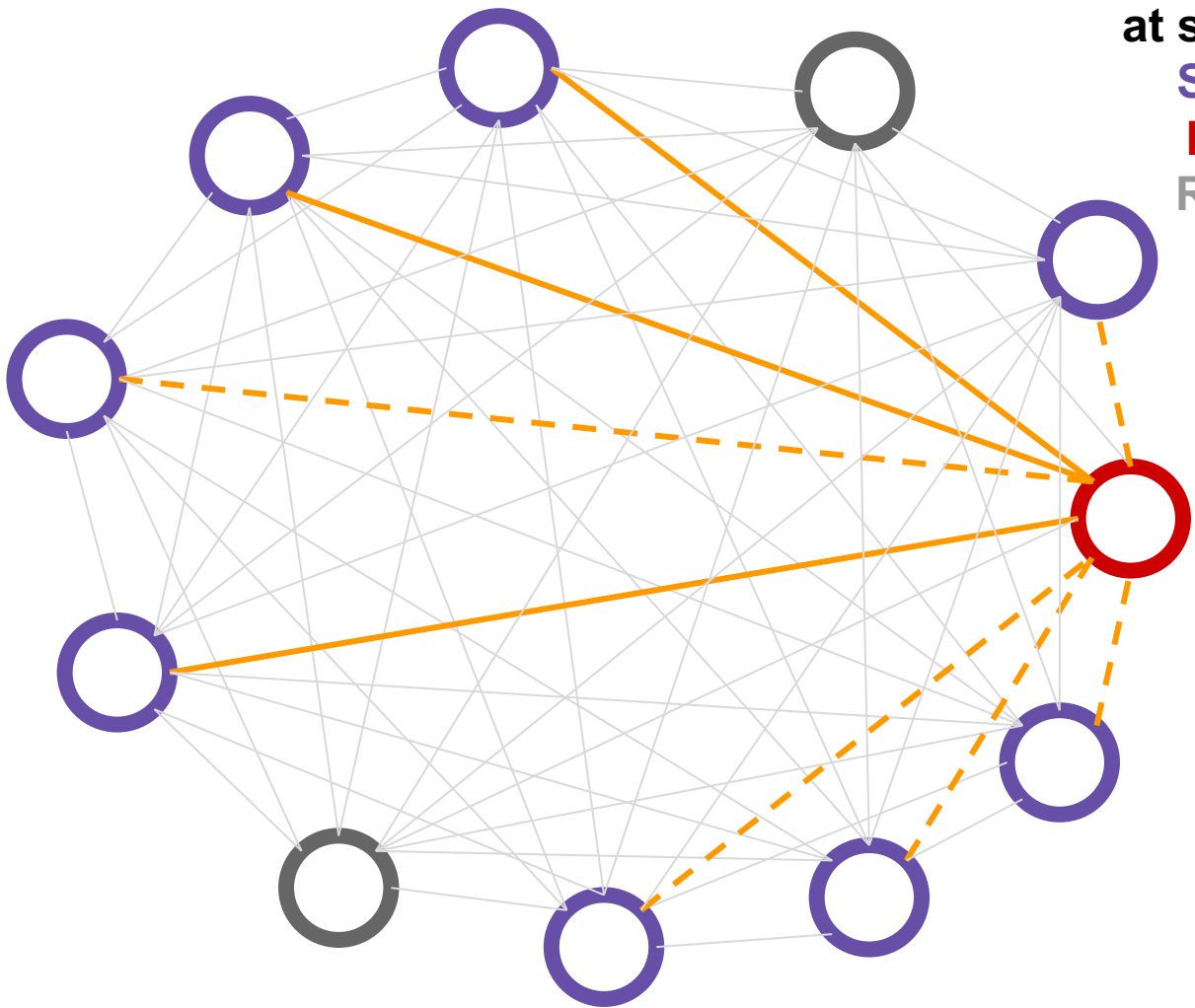
$R = 2$

at some t ,

$S = 8$

$I = 1$

$R = 2$



Population, N , of hosts

“Time” in steps: $t, t+1, \dots$

Hosts either *Susceptible*, *Infectious*, or *Removed*

Each time step, *all neighbours* interact

When I and S interact at t , probability p that the S becomes I at $t+1$

All I hosts at a time step t become R at $t+1$

Population, N , of hosts

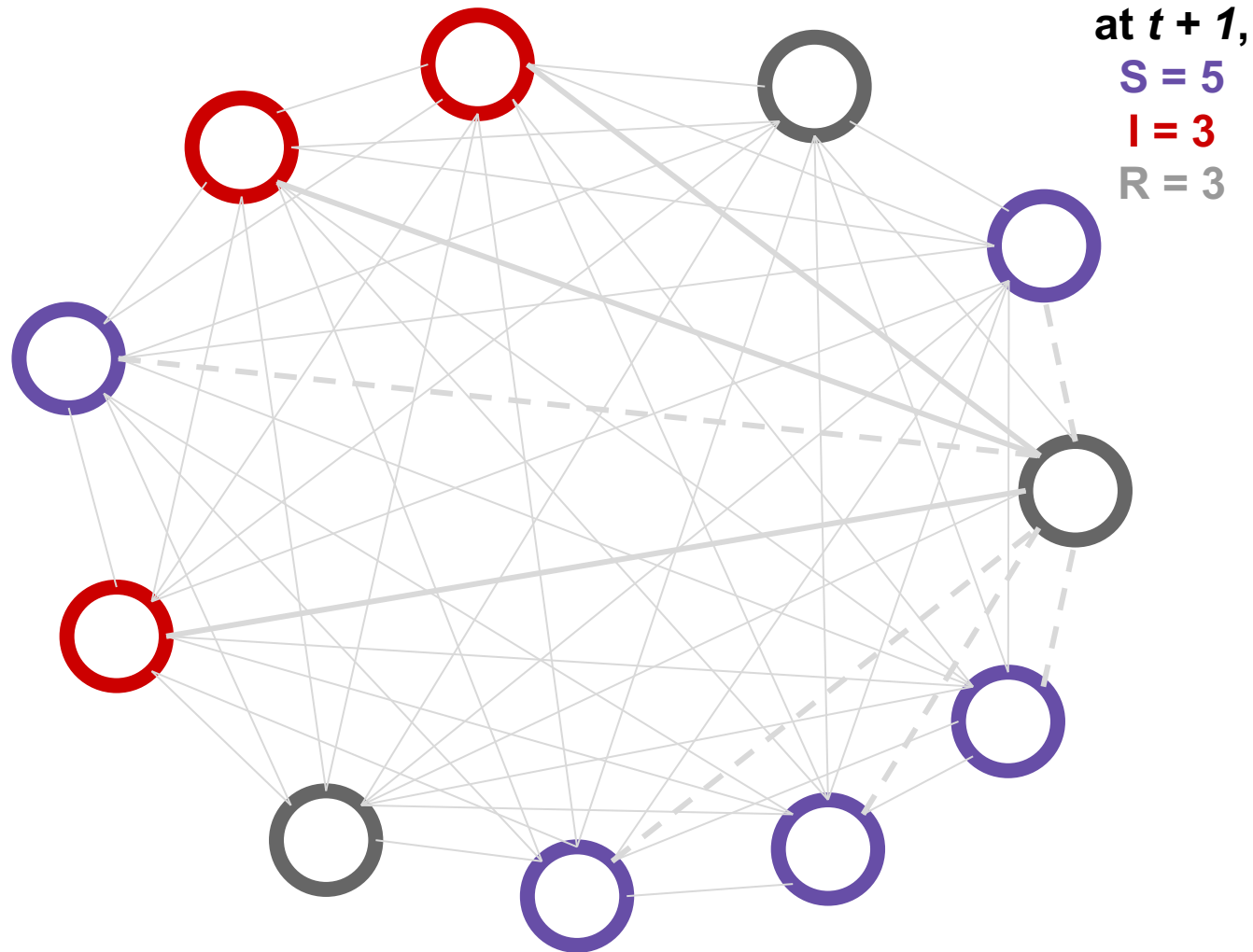
“Time” in steps: $t, t+1, \dots$

Hosts either *Susceptible*,
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All I hosts at a time step t
become R at $t+1$



Monkeypox virus, which causes mpox, spreads by close contact
Flu-like symptoms give way to rash especially on hands and feet,
anogenital region, throat, trunk

Clade I and Clade II differ in transmission characteristics and severity

The median incubation period for the 2022 Clade IIb outbreak was 9
days (IQR: 5–13 days; McFarland et al., Euro Surveill.¹)

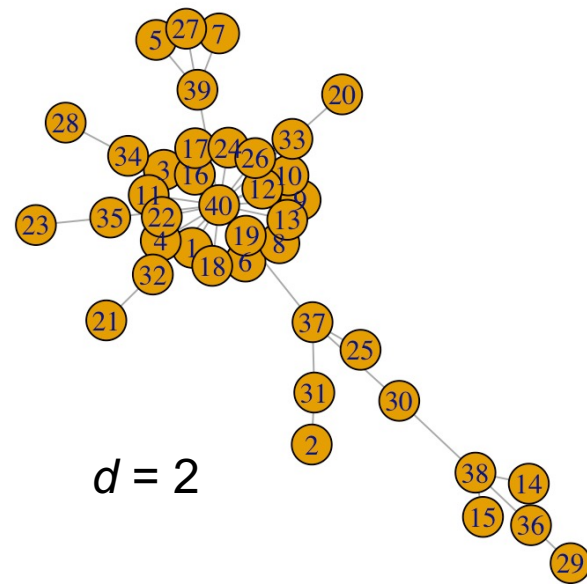
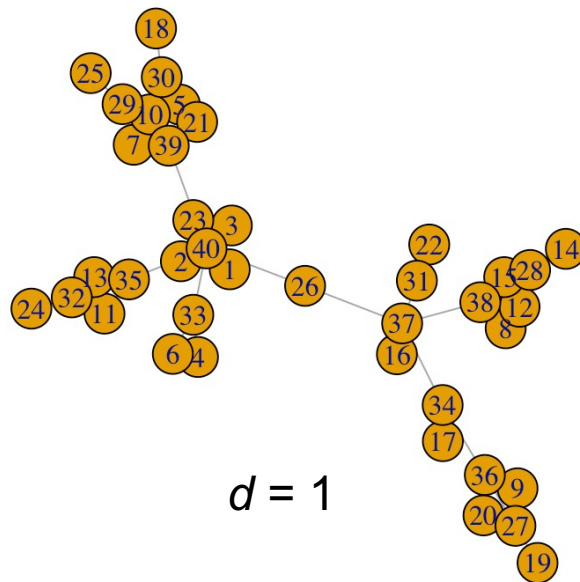
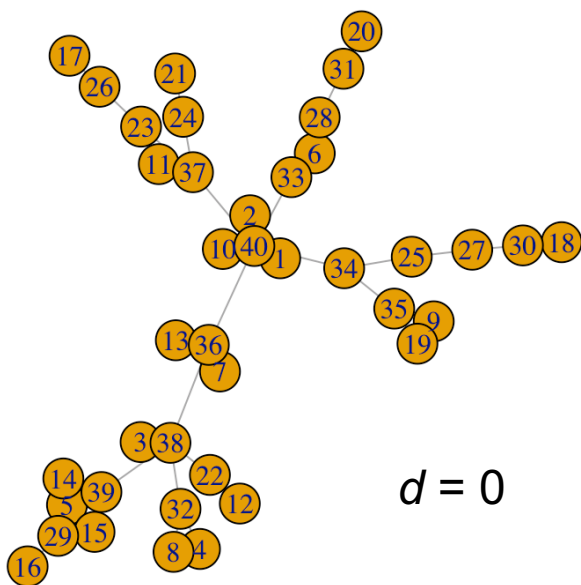
Transmission of Clade II has often been primarily among sexual
networks of GBMSM due to close contact

¹ <https://doi.org/10.2807/1560-7917.ES.2023.28.27.2200806>

A network model of mpox transmission

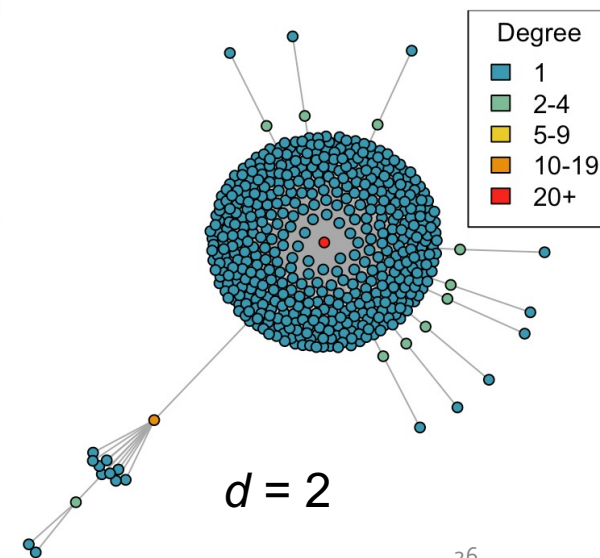
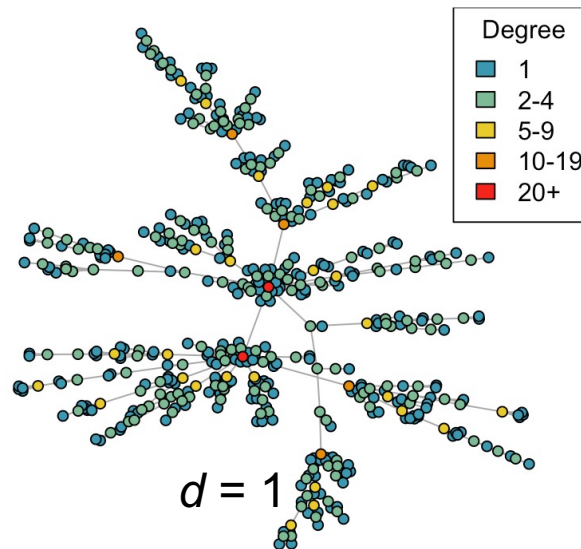
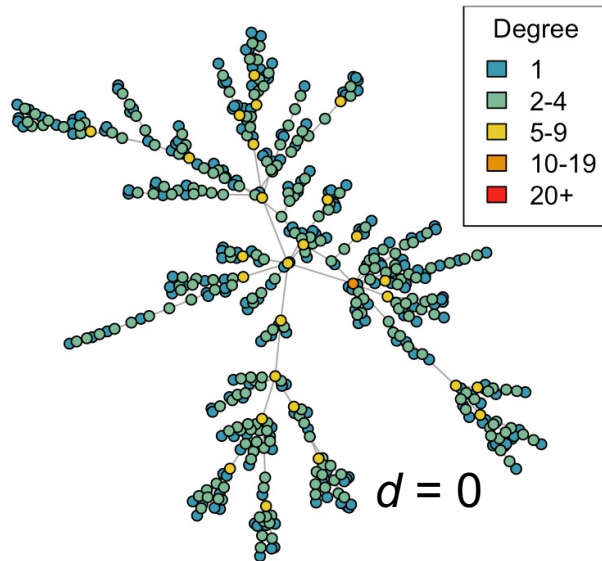
Questions from the practical

1. Try varying the d parameter for `create_network()` between 0 and 2. What changes about the network?



Questions from the practical

2. Plot using `plot_degree()` for 500 nodes...



Questions from the practical

```
net <- create_network(500, 1)
```

```
net <- network_step(net, p = 0.8)  
plot_state(net)
```

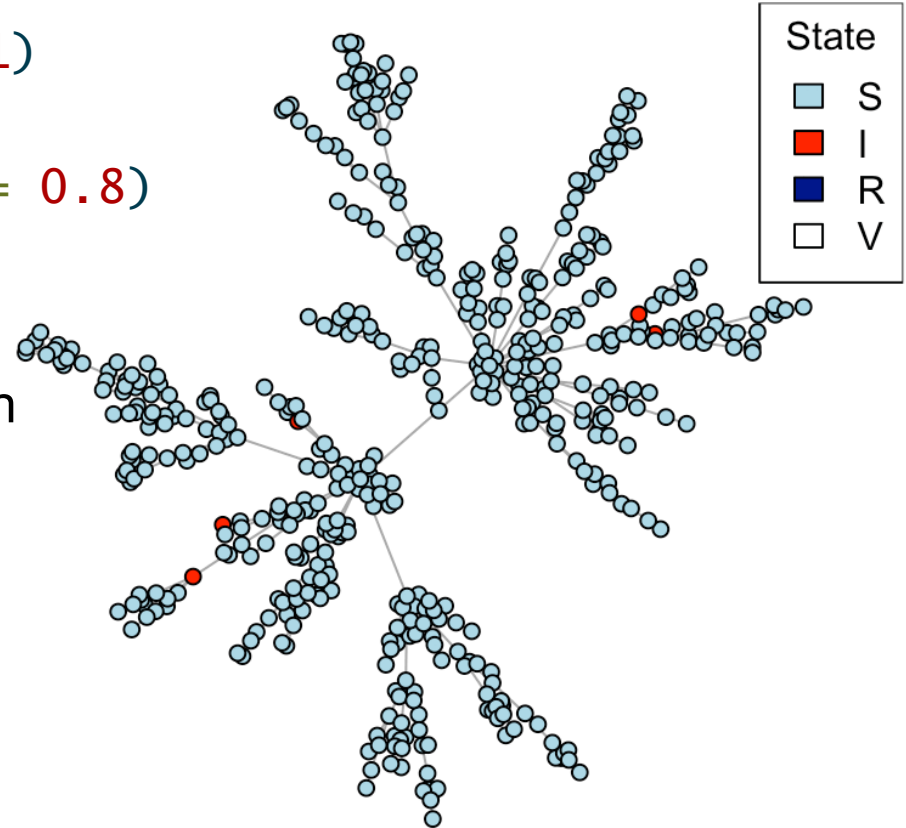
3. After creating your network with the first line above, run the last two lines repeatedly to watch the network model evolve.

Questions from the practical

```
net <- create_network(500, 1)
```

```
net <- network_step(net, p = 0.8)  
plot_state(net)
```

3. After creating your network with the first line above, run the last two lines repeatedly to watch the network model evolve.



Questions from the practical

4. How does the preferential attachment parameter (`create_network` parameter `d`) affect the final outbreak size?

Summary of 100 runs

<code>d</code>	Final outbreak size Median (90% CI)
0.0	207 (84–308)
1.0	261 (131–335)
2.0	397 (379–415)

(Tip: quicker to run if you remove the plotting and pausing statements from `run_mode1`.)

Further “bonus” examples in the practical explore introducing vaccination to the model, and explores the impact of targeting vaccination to the least-connected or to the most-connected individuals.

Introduced network modelling concepts

Discussed and implemented Reed-Frost **SIR** as a network model

Demonstrated strengths of network approach

Closing questions?

For early discussion of Reed-Frost Model, see e.g. *An examination of the Reed-Frost theory of epidemics.* Abbey H. *Hum Biol.* 1952 Sep; 24(3):201-33.